

Course Code: 23BS2T04

BONAM VENKATA CHALAMAYYA INSTITUTE OF TECHNOLOGY &
SCIENCE
(AUTONOMOUS)

I - B. TechII-Semester Supplementary Examinations (BR23), Sep/Oct - 2024
Differential Equations & Vector Calculus (All Branches)

Time: 3 hours

Max. Marks: 70

Question Paper consists of Part-A and Part-B
Answer ALL the question in Part-A and Part-B

PART-A (10X2 = 20M)

| | Marks | CO | BL |
|--|-------|------|----|
| 1. a) Solve the differential equation $x dy - y dx = a(x^2 + y^2)dy$ | (2M) | CO 1 | L3 |
| b) State Newton's Law of Cooling. | (2M) | CO 1 | L3 |
| c) Define Linear differential equations in y | (2M) | CO 2 | L1 |
| d) Find Particular Integral of $(D^3 - 5D^2 + 8D - 4)y = e^{2x}$ | (2M) | CO 2 | L1 |
| e) Form the partial differential equation by eliminating the arbitrary constants from $2z = \sqrt{x+a} + \sqrt{y+b}$ | (2M) | CO 3 | L1 |
| f) Solve $[D^3 - 3D^2D' + (D')^3]z = 0$. | (2M) | CO 3 | L3 |
| g) Define directional derivative of Scalar point function | (2M) | CO 4 | L1 |
| h) Prove that $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational | (2M) | CO 4 | L3 |
| i) State Gauss divergence theorem. | (2M) | CO 5 | L4 |
| j) If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$ evaluate $\int_C \vec{F} d\vec{r}$ where C is the curve $y = 2x^2$ in the xy-plane from (0,0) to (1,2) | (2M) | CO 5 | L6 |

PART-B (5X10 = 50M)

| | | | |
|--|-------|------|----|
| 2.a) (i) Solve the differential equation $(x^4 + y^4)dx - xy^3dy = 0$. | 5(M) | CO 1 | L3 |
| (ii) Solve the differential equation $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$ | 5(M) | CO 1 | L3 |
| (OR) | | | |
| b) If the temperature of the body is changing from 100°C to 70°C in 15 minutes, find when the temperature will be 40°C, if the temperature of air is 30°C. | 10(M) | CO 1 | L3 |
| 3.a) (i) Solve the differential equation $y^{11} - 4y^1 + 3y = 4e^{3x}$, given that $y(0)=-1$, $y^1(0)=3$ | 5 (M) | CO 2 | L4 |
| (ii) Solve the differential equation $(D^2 - 2D + 1)y = x^2e^{3x} - \sin 2x + 3$ | 5 (M) | CO 2 | L4 |
| (OR) | | | |
| b) Solve the differential equation $(D^2 + a^2)y = \sec ax$ using method of variation of | 10(M) | CO 2 | L4 |

parameters.

4.a) (i) Form the partial differential equation by eliminating an arbitrary function from $z = xy + f(x^2 + y^2)$. 5 (M) CO 3 L1

(ii) Find Solution of $px^2 + qy^2 = z^2$. 5 (M) CO 3 L3

(OR)

b) Solve $\frac{\partial^2 z}{\partial x^2} - 6 \frac{\partial^2 z}{\partial x \partial y} + 9 \frac{\partial^2 z}{\partial y^2} = 12x^2 + 36xy$. 10(M) CO 3 L3

5.a) (i) Find the Directional derivative of a scalar point function $\varphi(x, y, z) = 4xy^2 + 2x^2yz$ at the point A(1,2,3) in the direction of the line AB, where B=(5,0,4) 5(M) CO 4 L1

(ii) Prove that $\text{div}(\text{grad } r^m) = m(m+1)r^{m-2}$ 5(M) CO 4 L4

(OR)

b) Find constants a, b, c so that the vector $A = (x+2y+az) \mathbf{i} + (bx-3y-z) \mathbf{j} + (4x+cy+2z) \mathbf{k}$ is irrotational. Also find ϕ such that $A = \nabla\phi$. 10(M) CO 4 L1

6.a) (i) If $\vec{F} = (5xy - 6x^2)\mathbf{i} + (2y - 4x)\mathbf{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the curve C in the y-plane $y = x^3$ from (1, 1) to (2, 8). 5(M) CO 5 L6

(ii) Evaluate $\int_S \vec{f} \cdot \vec{n} ds$ where $\vec{f} = z\mathbf{i} + x\mathbf{j} - 3y^2z\mathbf{k}$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between $z=0$ and $z=5$. 5(M) CO 5 L6

(OR)

b) Verify Green's theorem for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region bounded by $x = 0, y = 0$ and $x+y=1$ 10(M) CO 5 L3
