

NUMERICAL METHODS AND COMPLEX VARIABLES
(Electrical and Electronics Engineering)

Time: 3 hours

Max. Marks: 70

Question paper consists of Part A & Part B
 Answer **All** the questions in Part A & Part B

Part A(10 X 2 = 20M)

1.a	Find the first approximation of $e^x \sin x = 1$ by using Bisection method.	[2M]	CO1	L2								
b	Prove that $\mu^2 = 1 + \frac{\delta^2}{4}$.	[2M]	CO1	L3								
c	Find $\int_0^2 f(x)dx$ from the following data and using Trapezoidal rule.	[2M]	CO2	L2								
	<table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tbody> <tr> <td style="padding: 2px 10px;">x</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0.5</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">1.5</td> <td style="padding: 2px 10px;">2</td> </tr> <tr> <td style="padding: 2px 10px;">$f(x)$</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0.25</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">2.25</td> <td style="padding: 2px 10px;">4</td> </tr> </tbody> </table>				x	0	0.5	1	1.5	2	$f(x)$	0
x	0	0.5	1	1.5	2							
$f(x)$	0	0.25	1	2.25	4							
d	Write RK method of 2 nd order formula.	[2M]	CO2	L1								
e	Define analytic function and entire function.	[2M]	CO3	L1								
f	State Cauchy's Integral Formula.	[2M]	CO3	L1								
g	Find the residue of the function $f(z) = \frac{2z+4}{(z+1)(z^2+1)}$ at $z = i$.	[2M]	CO4	L2								
h	Write the Laurent series of the function $f(z)$ having the center at $z = a$.	[2M]	CO4	L1								
i	Define Conformal Mapping.	[2M]	CO5	L1								
j	Find the image of the circle $ z = 2$, under the transformation $w = z + 3 + 2i$.	[2M]	CO5	L2								

Part B (5 X 10 = 50)

2.a.	Find a real root of the equation $x \log_{10} x - 1.2 = 0$ using Regula-falsi method.	5(M)	CO1	L2						
b.	Using Lagrange's interpolation formula, find the value of $y(10)$ from the following table:	5(M)	CO1	L2						
	<table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tbody> <tr> <td style="padding: 2px 10px;">x</td> <td style="padding: 2px 10px;">5</td> <td style="padding: 2px 10px;">6</td> <td style="padding: 2px 10px;">9</td> <td style="padding: 2px 10px;">11</td> </tr> <tr> <td style="padding: 2px 10px;">y</td> <td style="padding: 2px 10px;">12</td> <td style="padding: 2px 10px;">13</td> <td style="padding: 2px 10px;">14</td> <td style="padding: 2px 10px;">16</td> </tr> </tbody> </table>				x	5	6	9	11	y
x	5	6	9	11						
y	12	13	14	16						

OR

3	a	Evaluate $\sqrt{28}$ to four decimal places using Newton-Raphson method.	5(M)	CO1	L3										
	b	Find the Newton's Forward difference interpolating polynomial for the following data and hence find $f(2.5)$ from the polynomial. <table border="1" style="margin-left: 20px;"> <tbody> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>$y = f(x)$</td> <td>1</td> <td>3</td> <td>7</td> <td>13</td> </tr> </tbody> </table>	x	0	1	2	3	$y = f(x)$	1	3	7	13	5(M)	CO1	L2
x	0	1	2	3											
$y = f(x)$	1	3	7	13											
4	a	Evaluate $\int_0^6 \frac{1}{1+x} dx$ by using Simpson's 3/8th rule.	5(M)	CO2	L3										
	b	Given that $\frac{dy}{dx} = 1 + xy$ and $y(0) = 1$, compute $y(0.1)$ and $y(0.2)$ using Picard's method	5(M)	CO2	L3										
OR															
5	a	Find $y(0.1)$ from the differential equation $y' = x - y^2$, $y(0) = 1$ by using Taylor's series method	5(M)	CO2	L2										
	b	Use Runge-Kutta method of fourth order to evaluate $y(0.1)$ and given that $y' = x + y$, $y(0) = 1$.	5(M)	CO2	L3										
6	a	If $f(z)$ is a regular function of z , then prove that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] f(z) ^2 = 4 f'(z) ^2$	5(M)	CO3	L5										
	b	Find the analytic function whose imaginary part is $e^x(x \sin y + y \cos y)$.	5(M)	CO3	L2										
OR															
7	a	Show that the function $u(x, y) = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate.	5(M)	CO3	L2										
	b	Evaluate $\int_c \frac{e^{2z}}{(z-1)(z-2)} dz$ where c is the circle $ z = 3$.	5(M)	CO3	L3										
8	a	Obtain the Taylor's series expansion to represent the function $\frac{z^2-1}{(z+2)(z+3)}$ in the region $ z < 2$.	5(M)	CO4	L3										
	b	Evaluate by residue theorem $\int_c \frac{ze^z}{(z^2+9)} dz$, where c is the circle $ z = 5$.	5(M)	CO4	L3										
OR															
9	a	Obtain the Laurent's series of the function $\frac{z+3}{z(z^2-z-2)}$ in powers of z where $1 < z < 2$	5(M)	CO4	L3										
	b	Show that by method of residues $\int_0^\pi \frac{d\theta}{a+b \cos \theta} = \frac{\pi}{a^2-b^2}$, $a > b > 0$.	5(M)	CO4	L2										
10	a	Show that the function $w = \frac{4}{z}$ transforms the straight line $x = c$ in the z -plane into a circle in the w -plane.	5(M)	CO5	L2										
	b	Find the bilinear transformation which maps the points $\infty, i, 0$ in the z -plane into $-1, -i, 1$ in the w -plane.	5(M)	CO5	L2										
OR															
11	a	Show that the transformation $w = z^2$ maps the circle $ z - 1 = 1$ into the cardioid $r = 2(1 + \cos \theta)$ where $w = re^{i\theta}$ in w -plane.	5(M)	CO5	L2										
	b	Find the fixed points of the following transformations. (i). $w = \frac{2i-6z}{iz-3}$ (ii). $w = \frac{6z-9}{z}$	5(M)	CO5	L2										