Course Code: 23BS1T02

BONAM VENKATA CHALAMAYYA INSTITUTE OF TECHNOLOGY & SCIENCE

(AUTONOMOUS)

I-B. Tech I-Semester Regular/Supplementary Examinations (BR23), January - 2026

LINEAR ALGEBRA & CALCULUS (Common to All Branches)

Time: 3 hours | Max. Marks: 70

Question Paper consists of Part-A and Part-B
*Answer **ALL** the question in **Part-AandPart-B***

PART–A (10X2 = 20M)

		Marks	CO	BL
1. a)	Define the rank of a matrix and give an example	(2M)	CO1	L1
b)	For what values of h, the equations $x + y + z = 1; x + 2y + 4z = h;$ $x + 4y + 10z = h^2$ are consistent	(2M)	CO1	L3
c)	Find the product of the eigen values of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$	(2M)	CO2	L1
d)	Identify the nature and signature of the Q.F $x^2 + 2y^2 + 3z^2$	(2M)	CO2	L3
e)	Determine the intervals in which the polynomial $f(x) = (4 - x^2)^2$ is increasing or decreasing.	(2M)	CO3	L5
f)	Find the Maclaurin's series expansion of $f(x) = e^x$.	(2M)	CO3	L1
g)	Find the first order partial derivatives $z = x^3 + y^3 - 3axy$ with respective to x and y	(2M)	CO4	L1
h)	If $u = tan^{-1}\left(\frac{y}{x}\right)$ then find $xu_x + yu_y$ by using Euler's theorem.	(2M)	CO4	L3
i)	Evaluate $\int_1^2 \int_1^3 xy^2 \, dx \, dy$.	(2M)	CO5	L5
j)	Evaluate $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2) \, dx \, dy \, dz$.	(2M)	CO5	L5

PART–B (5X10 = 50M)

2.a)	Reduce the following matrix A to normal form and hence find the rank of the matrix. $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$	5(M)	CO1	L1
b)	Determine the values of a and b for which the system $x + 2y + 3z = 6;$ $x + 3y + 5z = 9; 2x + 5y + az = b$ (i) no solution (ii) a unique solution (iii) an infinite number of solutions.	5(M)	CO1	L5
	(OR)			
3.a)	Reduce the matrix A to Echelon form and hence find the rank of the matrix. $A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$	5(M)	CO1	L1
b)	Solve the system of equations using Gauss-Seidel method $5x + 2y + z = 12$ $x + 4y + 2z = 15$ $x + 2y + 5z = 20$	5(M)	CO1	L3

4.	State Cayley-Hamilton theorem and verify the theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Hence evaluate $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$	10(M)	CO2	L5
	(OR)			
5.	Find the eigen values and eigen vectors for the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.	10(M)	CO2	L1
6.a)	If $a < b$, prove that $\frac{b-a}{(1+b^2)} < tan^{-1}b - tan^{-1}a < \frac{b-a}{(1+a^2)}$ using Lagrange's Mean value theorem. And hence deduce that $\frac{\pi}{4} + \frac{3}{25} < tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.	5(M)	CO3	L3
b)	Verify Rolle's theorem for $f(x) = log_e\left(\frac{x^2+ab}{x(a+b)}\right)$ in $[a \quad b]$ where $a, b > 0$	5(M)	CO3	L4
	(OR)			
7.a)	Verify Cauchy's mean value theorem for the functions $f(x) = \ln x$ and $g(x) = \frac{1}{x}$ in $[1\ ,\ e]$.	5(M)	CO3	L4
b)	For every $x \geq 0$, show that $1 + x + \frac{x^2}{2} \leq e^x \leq 1 + x + \frac{x^2}{2}e^x$.	5(M)	CO3	L3
8.a)	If $u = \frac{x+y}{1-xy}$, $v = tan^{-1}x + tan^{-1}y$ then show that u, v are functionally related and find the relationship.	5(M)	CO4	L3
b)	Discuss the maxima and minima of $x^3y^2(1 - x - y)$.	5(M)	CO4	L6
	(OR)			
9.a)	If $u = x^2 - y^2, v = 2xy$ and $x = rcos\,\theta, y = r\sin\theta$ then find $\frac{\partial(u,v)}{\partial(r,\theta)}$.	5(M)	CO4	L3
b)	Determine the point on the plane $2x + 3y - z = 5$ which is nearest to the origin.	5(M)	CO4	L5
10.a)	Change of order of integration and evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dxdy$.	5(M)	CO5	L5
b)	Evaluate $\iint_R r sin\,\theta\, dr\, d\theta$ over the cardioid $r = a(1 - \cos\theta)$ above the initial line.	5(M)	CO5	L5
	(OR)			
11.a)	Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16\,a^2}{3}$.	5(M)	CO5	L3
b)	Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ by changing into polar coordinates.	5(M)	CO5	L5

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